every third number after 3, since each of these numbers is divisible by 3. This produces row 3 of Table 3.6.

Continuing in this way, we are left with only the primes remaining. In the example above, the numbers remaining are 2, 3, 5, 7, 11, and 13 and these are all the primes less than 15. Given enough paper (or papyrus) and enough time, it is straightforward to write long lists of primes.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
-	2	3	-	5	-	7	-	9	-	11	-	13	-	15
-	2	3	-	5	-	7	-	-	-	11	-	13	-	-

Table 3.6: The Sieve of Eratosthenes is used to compute prime numbers. The first row shows the sieve at the start, and the second row shows the sieve after 1 and all multiples of 2 are removed. The third row shows the sieve after all multiples of 3 are removed.

It is easy to write a computer program that implements this algorithm. For example, in the notes there is an eleven line Python program that computes primes in this way. Using this program, I can compute on my laptop all the primes less than or equal to $250,000$ in a second or so. By the way, there are $22,044$ of them — try it.

If all the primes smaller than a fixed number are needed — say all the primes less than $1,000,000$ — then the Sieve Method (and its variants) is still the best way to find them. This is not bad for an algorithm that is over 2000 years old.

Mersenne Primes. Numbers of the form

$$M_n = 2 \times 2 \times 2 \cdots \times 2 - 1, \qquad \text{(2 occurs } n \text{ times)}$$

are called *Mersenne Numbers.* If a Mersenne number M_n is prime, it is called a *Mersenne Prime.* For a small prime numbers n, Mersenne Numbers are prime, which led to the mistaken belief that they are prime for all n. They are named after Marin Mersenne, who was a French monk